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CELLULAR CONVECTION IN A SALINITY GRADIENT ALONG A HEATED INCLINED WALL

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Abstract—Natural convection along a heated inclined wall in a salinity gradient has been studied experimentally. The experiments were carried out in a tank with one wall inclined at 45°. The dimensions of the tank were 20 cm in height and depth, 30 cm wide at the top, and 10 cm wide at the bottom. The inclined wall and the opposite vertical wall were made of aluminum and they were kept at different constant temperatures. The other two walls were of plexiglass. Shadowgraph technique was used to visualize the fluid motion. The onset of celular motion was quite similar to the case of heating along a vertical wall. In the subcritical regime, the fluid moved along the heated wall almost imperceptibly. In the transition regime, convection cells were generated successfully from the bottom boundary. In the supercritical regime, simultaneous occurrence of the convection cells were observed. These cells then grew clear across the tank. The motions in the fully developed cells were quite different from those observed in tanks with vertical walls. The governing parameter which characterizes the fluid motion is the Rayleigh number defined in terms of the height of rise of a heated fluid particle and the reduced gravity. It was found that the Rayleigh number range for the transitional regime is somewhat higher than that for the vertical heated wall. This was due to the stabilizing effect of the additional hydrostatic pressure normal to the wall. Longitudinal vortices along the inclined wall were observed only in a large bottom cell 6 days into the experiment. At that time, the fluid within the cell was of constant salinity. In no other case longitudinal vortices were observed.

NOMENCLATURE

g, gravitational acceleration [cm/s^2];
h, natural length scale $= \alpha\Delta T/\phi_0$[cm];
R, Rayleigh number $= g\cos\theta\alpha\Delta T\, h^3/\kappa\nu$;
T_{amb}, ambient temperatire [°C];
ΔT, temperature difference [°C].

Greek symbols

α, coefficient of thermal expansion [°C^{-1}];
θ, angle of inclination from vertical;
κ, thermal diffusivity [cm^2/s];
ν, kinematic viscosity [cm^2/s];
ρ, density [gm/cm^3];
ϕ_0, initial density gradient

$$= -\frac{1}{\rho}\frac{\partial\rho}{\partial z}[\text{cm}^{-1}].$$

1. INTRODUCTION

THE MOTION of a stratified salt solution being heated laterally from a vertical wall has been investigated by Thorpe, Hutt and Soulsby [1], Chen, Briggs and Wirtz [2], and Wirtz, Briggs and Chen [3]. For a given density gradient, at small temperature differences ΔT between the heated wall and the fluid, the motion of the fluid due to buoyancy is parallel to the heated wall at almost imperceptible speeds. Due to the boundary effects of the top and bottom walls, a small number of horizontal convection cells are generated there. This is the subcritical regime. As ΔT is increased into the transition regime, horizontal convection cells grow successively inward from the bottom boundary. This phenomenon was first noticed ninety years ago by Brewer [4] in the form of stratified subsidence of fine particles in water. Mendenhall and Mason [5] demonstrated clearly that this was due to the presence of lateral temperature gradients. In the supercritical regime, at still higher ΔT, convection cells occur simultaneously all along the heated wall. A Rayleigh number based on the natural length scale which is the height of rise of a heated fluid particle in the undisturbed stratified surrounding was shown to be the governing parameter in [2]. We have also determined that the transition from the subcritical to the supercritical flow occurs in a Rayleigh number range 12500–17500.

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When heating is supplied along an inclined wall, if the angle of inclination is small, the principal effect is expected to be that of reduced gravity. However, when this angle exceeds 30° from the vertical, Sparrow and Husar [6] have found the existence of longitudinal vortices along the inclined plate in a homogeneous fluid. The number of vortices was observed to increase with increasing ΔT. Within the range of 30–60° from the vertical, the angle of inclination of the plate didn't have any effect on the size or number of vortices.

In [2] and [3], we had speculated that the microstructures, or the layer structures, observed in many parts of the world's oceans may owe their existence, at least in part, to lateral heating from the continental boundaries. Since these boundaries are seldom vertical, one then wonders whether layer convection may exist along a sloping heated wall. If longitudinal vortices do exist in a stratified fluid, they may introduce three-dimensional motion large enough to disrupt any cellular convection.

To answer the question raised above, we have undertaken a simple experiment of natural convection of a stratified fluid along a heated inclined wall. The experiment was performed in a small tank with one wall inclined at 45°. It is clear that the angle of inclination must be between 30° and 60°, a range in which longitudinal vortices most likely occur. With a 45° inclination, the components of gravitational vector are the same in the directions parallel and normal to the heated surface. The effect of a hydrostatic pressure normal to the plate on the onset of secondary flow may be explored. For angles which exceed 45° from the vertical, the effect of heating from below may become dominant. It is known [7] that stratified fluid being heated from below will also generate horizontal convecting layers.

Preliminary tests showed that at sufficiently high ΔT, simultaneous occurrence of convection rolls were obtained all along the inclined wall. This is illustrated in Fig. 1 in which shadowgraphs for a $R = 1{\cdot}14 \times 10^5$ case are shown. It was then our

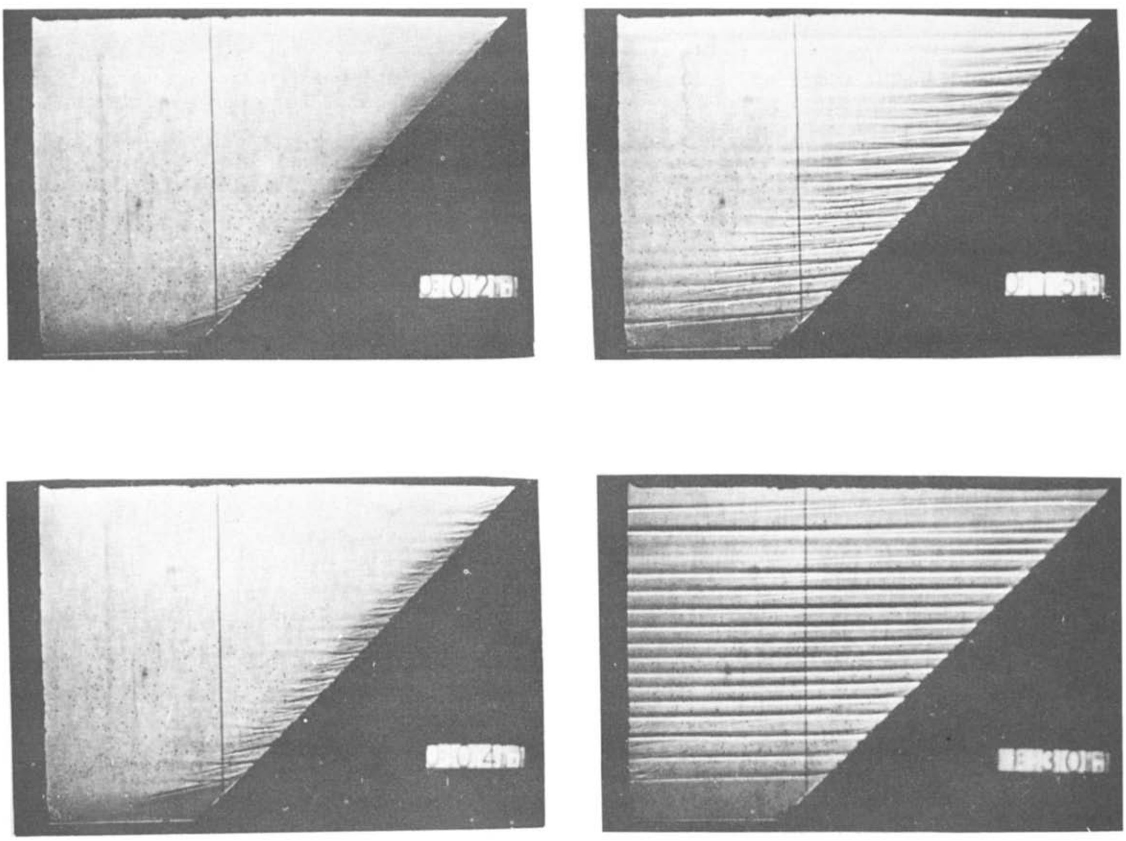

FIG. 1. Simultaneous occurrence of convection rolls along the heated inclined wall at a very high Rayleigh number of 114000.

purpose to carry out sufficient number of experiments to delineate the three flow regimes as encountered in [2] and [3], and to examine the flow in fully developed cells. The apparatus used and procedure taken for the experiments are described in § 2. Results and discussion are presented in § 3.

2. EXPERIMENTAL APPARATUS AND PROCEDURE

The experiments were conducted in a small tank with two aluminum and two plexiglass walls, resting on a plexiglass bottom wall. One aluminum wall was vertical and the other was inclined at 45° from the vertical. These two walls were separated by 10 cm at the bottom and 30 cm at the top; the height of the tank was 20 cm. The two plexiglass walls were separated by 20 cm. The two aluminum walls were provided with counter-flow passages through which water at preset temperature could be circulated. Two Haake constant temperature baths were used; one to maintain the hot inclined wall at a constant temperature and the other to keep the cold wall at the constant ambient temperature. In order to remove the heat received at the cold wall, the fluid in the Haake bath for the cold wall was cooled by an external cooling unit. The distance between the two aluminum walls was chosen wide enough so that the presence of the opposite wall did not affect the onset of convection rolls at supercritical Rayleigh numbers. The present test tank is a modification of the one used in previous experiments reported in [2] and [3].

To obtain a density stratified fluid with constant gradient, successive layers of salt solution with decreasing salinity were admitted slowly into the tank. The layers were 2·5 cm in depth. After filling, the tank was left to stand for 24 h to permit diffusion to smooth out the stepwise salinity distribution. Prior to the start of the experiment, the vertical salinity distribution was recorded using a conductivity probe. Then both baths were adjusted to the desired temperatures. Valves in the lines between the baths and their respective aluminum walls were then opened to commence the experiment. The surface temperatures of both aluminum walls at the mid-height were recorded continuously. It took about two minutes for the hot wall to reach 90 per cent of its final steady value. Fluid motion was made visible by means of shadowgraph. Events were recorded by photographs. Dye traces were used to discern motion in the convecting cells and to detect the presence of longitudinal vortices. A plumb line was suspended in front of the tank to serve as a reference for angles of inclination of the wall as well as the cell boundary.

To test out the system, a series of experiments were run with water. Five tests were run with ΔT ranging from 3°C to 13°C. Longitudinal vortices were observed in all cases, and the number of vortices per unit span was observed to increase with increasing ΔT. Because of the presence of the bottom wall, the vortices were seen to become turbulent in a short distance from the corner. We were also concerned about the possible presence of diffusive flow induced by the bending of constant density lines near the inclined wall as observed by Phillips [7]. Such motion was never observed in our experiments prior to the heating of the inclined wall. This is probably due to the fact that our density gradients were about $\frac{1}{7}$ of that used in Phillips' experiment.

3. RESULTS AND DISCUSSION

A total of six tests were carried out with the initial density gradient held approximately constant at $2{\cdot}6 \times 10^{-3}\ \mathrm{cm}^{-1}$ (stability frequency of $0{\cdot}62\ \mathrm{s}^{-1}$) and ΔT varied from 3·8°C to 8·2°C. The test conditions and results are summarized in Table 1. In this Table, κ denotes thermal diffusivity, ν kinematic viscosity, α coefficient of thermal expansion, and ϕ_0 the initial density gradient. The natural length scale h is the height of rise of a heated fluid particle in the undis-

Table 1. Summary of experimental conditions and results

Test no.	$R \times 10^{-4}$	Cell size / h	$\phi_0 \times 10^3$ (cm^{-1})	ΔT (°C)	T_{amb} (°C)	Mean specific gravity	$\kappa \times 10^3$ (cm^2/s)	$\nu \times 10^2$ (cm^2/s)	$\alpha \times 10^3$ (°C^{-1})
1	0·369	Sub	2·670	3·83	21·3	1·030	1·450	0·980	0·286
2	1·138	Sub	2·665	4·10	28·1	1·027	1·477	0·831	0·336
3	1·800	Tran	2·670	5·55	21·7	1·030	1·450	0·970	0·290
4	2·600	Tran	2·640	5·13	27·5	1·028	1·472	0·835	0·333
5	3·475	1·16	2·665	6·61	21·7	1·030	1·450	0·956	0·290
6	11·380	1·01	2·660	8·24	23·9	1·029	1·458	0·955	0·308

turbed stratified surrounding, $h = \alpha \Delta T / \phi_0$. The Rayleigh number is defined in terms of h and reduced gravity

$$R = \frac{g \cos \theta \alpha \Delta T h^3}{\kappa \nu}$$

in which θ is the angle of inclination from the vertical. The test results are arranged in increasing Rayleigh number. Two of these were in the subcritical regime, two in the transition regime, and two in the supercritical regime. These are separately discussed below.

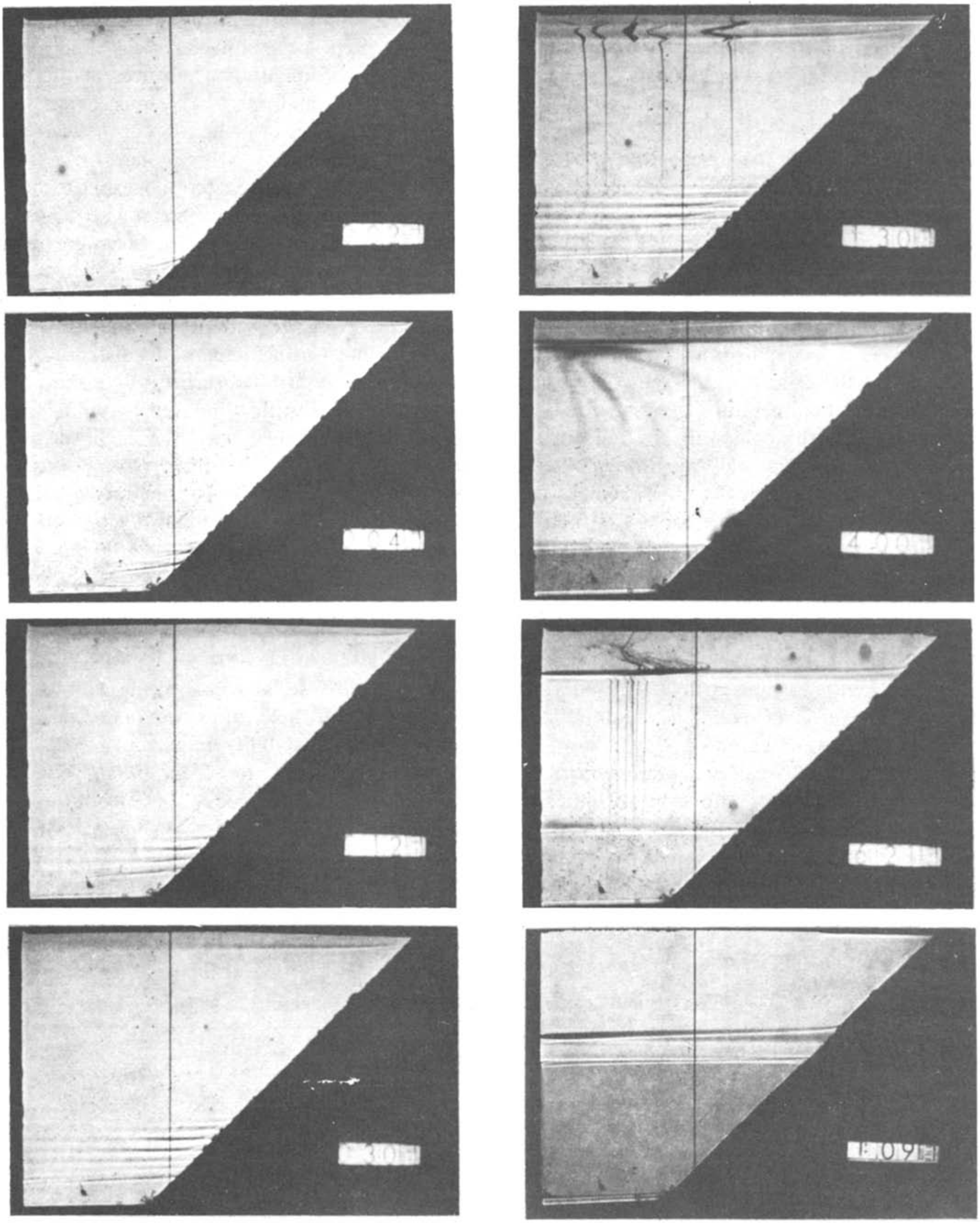

FIG. 2. Subcritical case, $R = 11\,380$.

Subcritical regime

This regime is characterized by a very slow process of successive generation of a small number of cells near the top and bottom wall. These cells eventually give way to two large cells one at the top and one at the bottom. These two cells, in turn, will grow in size and obliterate the more or less quiescent layer separating the two.

A series of photographs taken from test 2, $R = 1{\cdot}138 \times 10^4$ are shown in Fig. 2. At $t = 2$ min a cell is starting at the lower corner. By $t = 4$ min the number of cells has grown to three. Characteristically, the cells generated later have shorter lateral dimension. The number of cells becomes 4 at 8 min, 5 at 12 min, and attains the greatest number of 7 at 30 min. These are all confined to the lower $\frac{1}{3}$ of the tank. At this time, the bottom cell has reached the opposite vertical wall, and the top cell becomes quite visible. From this time on, the top and bottom cells slowly grow bigger and the interfaces advance downward and upward respectively. At the same time, cell erosion processes start to take effect in the cells above the bottom one. By 1 h 30 min, two cells immediately above the bottom one have disappeared. The erosion process which is encountered in all tests is probably a manifestation of instability of double-diffusive layers studied by Huppert [9]. He has shown that for a system of layers, if the ratio of salt flux to

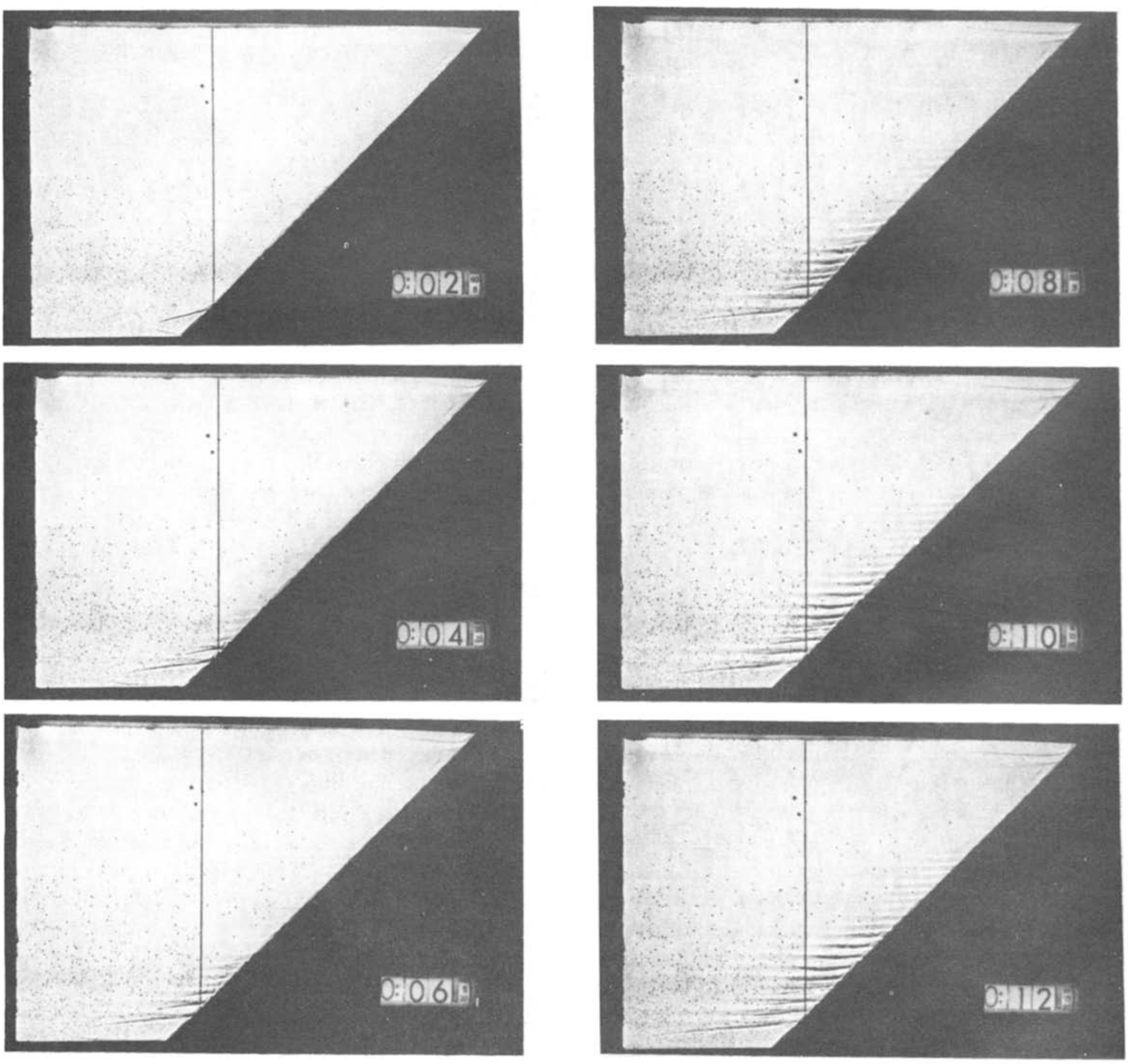

Fig. 3. Transitional case, $R = 18000$.

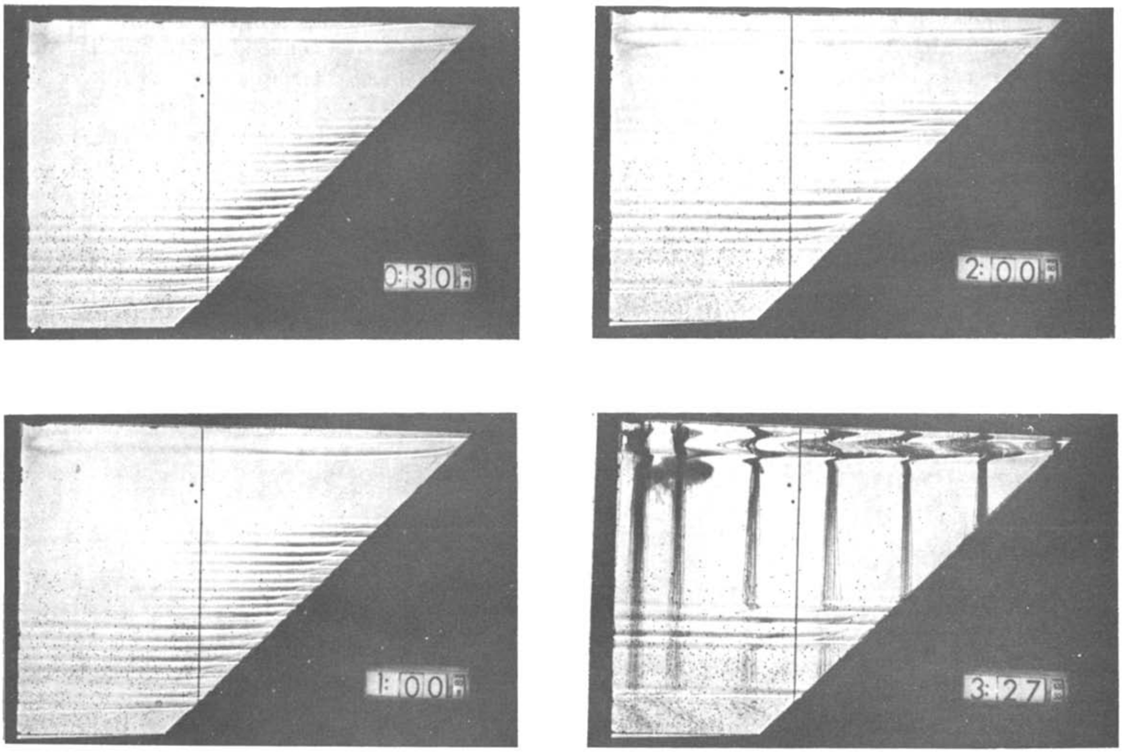

FIG. 3—*continued*

heat flux is constant across all interfaces, then the system is stable. Otherwise, the system is not stable and some layers will grow at the expense of their neighbors. At $t = 4$ h, there are no more intermediate layers. At 18 h 21 min, both the top and bottom cells have gained in size, and the dye streaks reveal that in the mid-section the motion is quite slow compared to that in the top layer. Six days later at $t = 145$ h 9 min, the middle layer has reduced to about 2 cm.

It was at this time, 6 days into the experiment, that the presence of longitudinal vortices were detected along the inclined wall in the bottom cell. Small air bubbles aligned themselves on the inclined wall in straight lines with clear regions in between. These bubbles accumulated in the regions between two adjacent counter-rotating vortices. Dye crystals dropped to the bottom of the tank would first dissolve into pools. The dye solution was then carried slowly toward the hot inclined wall. At the wall, they were stretched out into thin ribbons climbing up the wall slowly. Unfortunately, no satisfactory photographs were obtained. Of course at this time, the fluid in the lower cell was of uniform salinity. What we were seeing was the same phenomenon noted by Sparrow and Husar [6]. In no other tests did we detect the presence of longitudinal vortices.

Transitional regime

In this regime, the flow is characterized by a more rapid and extensive successive growth of cells along the inclined heated wall. This process is illustrated by the pictures taken from test 3, $R = 1{\cdot}8 \times 10^4$, as shown in Fig. 3. The rapid growth of the number of cells is evidenced by the photographs taken at 2 min intervals from 2 to 12 min. Starting with one cell at 2 min, it grows to 19 at 12 min and extends to $\frac{2}{3}$ of the tank. The lower ten cells are separated by interfaces with strong density gradients. The interfaces separating the upper nine cells are quite a bit weaker as evidenced by the faint bright lines in the shadowgraph. These interfaces are gradually sharpened by $t = 30$ min. By $t = 1$ h, erosion process has started in the cells immediately adjacent to the bottom one. One hour later, cells in two regions have disappeared. At $t = 3$ h 27 min, except for the top and bottom cells, there is only one region with two cells remaining in the mid-section of the tank. These will disappear and eventually only two cells will remain and the same process as shown in Fig. 2 will take place. It is interesting to note that in the region where cells have disappeared, the motion is much less noticeable than in the cellular convection layers as evidenced by the dye traces distributed all the way across the tank. The motion in the convecting cells is towards the hot wall along the bottom of the cell and away from the hot wall along the top of the cell.

At the higher transitional Rayleigh number of $2{\cdot}6 \times 10^4$ (Test 4) the process of successive generation of cells is the same except now it takes place more rapidly. By $t = 15$ min cells cover the entire tank. The transitional regime in the present 45° wall case extends approximately from $R = 1{\cdot}8$ to $2{\cdot}6 \times 10^3$. For vertical side walls, the regime extends from $R = 1{\cdot}25$ to $1{\cdot}75 \times 10^3$. This increase in transitional Rayleigh number range is due to the component of gravity normal to the inclined surface. At the onset of cellular motion, fluid must turn away from the wall and into

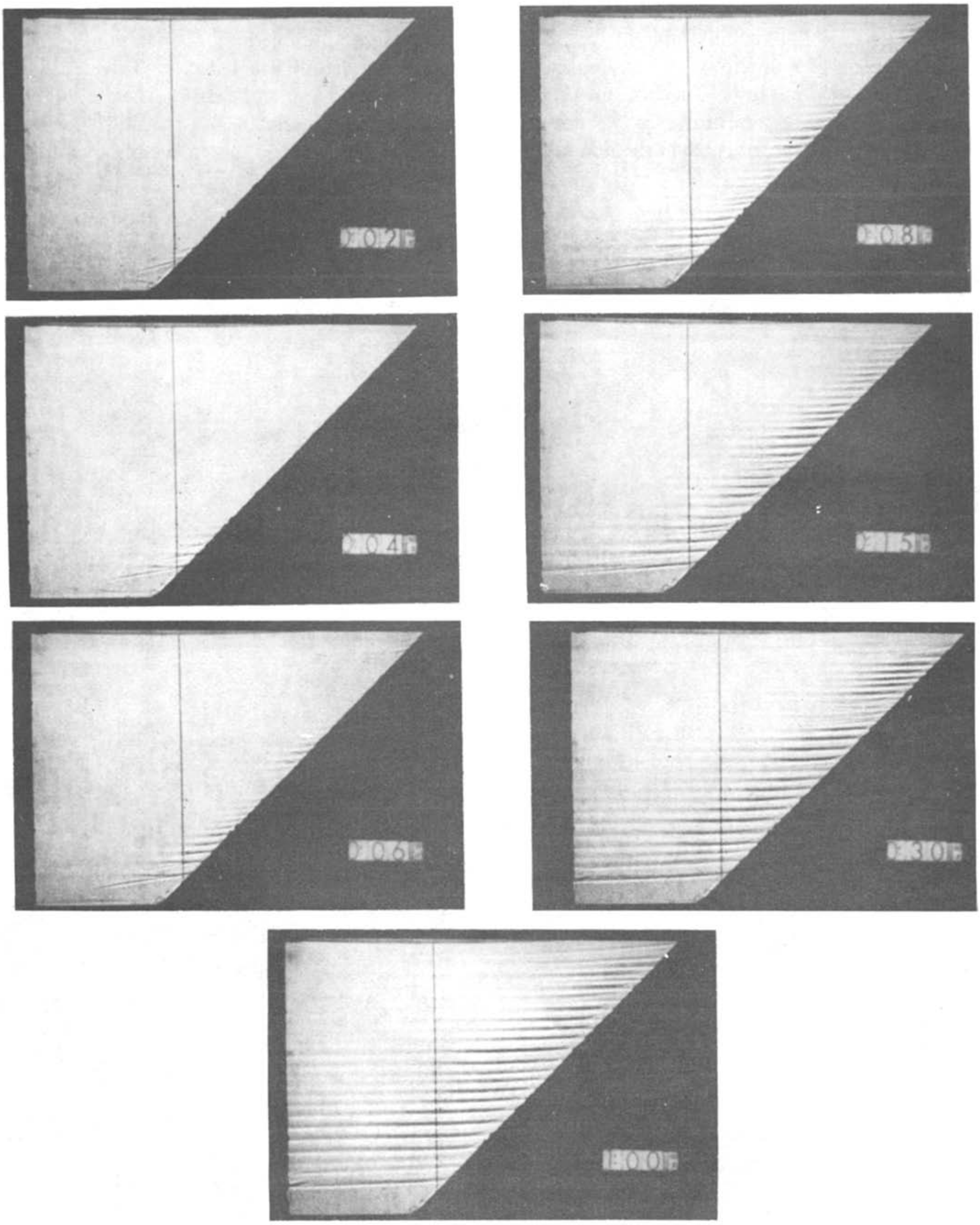

FIG. 4. Supercritical case, $R = 34750$.

the fluid. Now the additional normal hydrostatic pressure hinders such motion thus raising the transitional Rayleigh number.

Supercritical regime

This regime is characterized by the occurrence of convection cells along the heated wall in the mid-section of the tank away from the two ends. We use test 5, $R = 3{\cdot}475 \times 10^4$ to illustrate this case. The fluid motion at $t = 2$ min shown in Fig. 4 is exactly the same as in previous tests. The bottom cell has been initiated. At $t = 4$ min, however, we note the presence of two cells faintly visible in the mid-section of the tank. These two cells are not generated due to the boundary influences. The cells generated successively from the bottom has not yet reached the mid-section. The number of cells in the mid-section grows to five at $t = 6$ min. By $t = 15$ min, the entire heated inclined wall is covered with horizontal convection cells. These then develop into full circulation cells reaching clear across the tank as shown in the photographs taken at 30 min and 1 h. The average cell size obtained at $t = 20$ min for the two supercritical cases are listed in Table 1. They are almost exactly the same as the height of rise of a heated fluid particle in the stratified surrounding.

Cell erosion process then takes place at 8:52, there

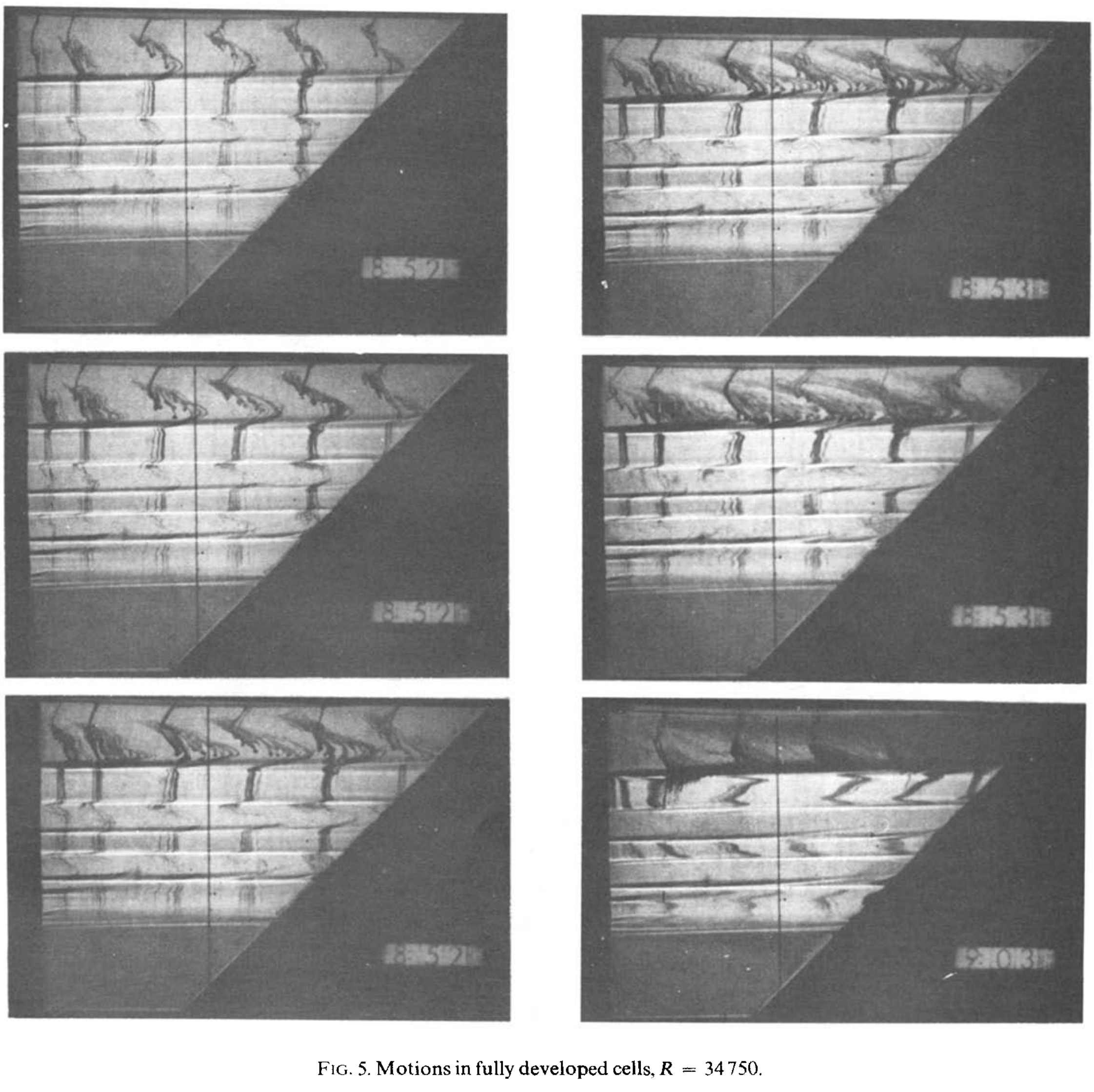

FIG. 5. Motions in fully developed cells, $R = 34\,750$.

remain seven cells. A series of five photographs taken of the dye streak approximately 18 s apart are shown in Fig. 5. These photographs were taken at 8:52:17, 8:52:34, 8:52:53, 8:53:11 and 8:53:30. Counter-clockwise circulations in the first, third and fifth cell from the top are clearly exhibited. The motion in these cells is so vigorous that by the end of the 73 s period the dye streaks are hardly visible. However, the motion in the second, fourth, and sixth cell from the top are very deliberate. At the end of the same time period, only slight deflections in the dye streaks are observed. After a 10 min interval, at 9:03, the dye streaks in these cells reveal a more complicated flow pattern. The motion along the bottom of the cell is toward the hot wall. At the hot wall, however, instead of following the wall up to the interface, the flow turns away from the hot wall immediately and flows towards the cold wall just above the lower layer. A similar situation prevails near the top interface. It seems that these cells are the result of merging of two or three cells. (The sixth cell from the top exhibits three of these reversals.) In the process of merging, the salinity becomes equalized but the velocity field in each cell persists. It remains a puzzle as to why an alternating pattern results.

4. CONCLUSIONS

A. When heating a stratified fluid from a 45° wall, the subcritical and the supercritical regimes are separated by a transitional regime which spans approximately from $R = 18\,000$ to $26\,000$. The increase of the critical Rayleigh number range over that for a vertically heated wall is attributed to the additional hydrostatic pressure normal to the wall, which retards the onset of the convection rolls.

B. Longitudinal vortices do not exist in natural convection of *stratified* fluid along an inclined wall. Once the density becomes uniform due to mixing, longitudinal vortices appear.

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CONVECTION CELLULAIRE DANS UN GRADIENT DE SALINITÉ LE LONG D'UNE PAROI CHAUDE ET INCLINÉE

Résumé—On a étudié expérimentalement la convection naturelle le long d'une paroi chaude et inclinée dans un gradient de salinité. Les expériences ont été menées dans un réservoir ayant une paroi inclinée à 45°. Ce réservoir a 20 cm de hauteur et de profondeur, 30 cm de largeur au sommet et 10 cm à la base. La paroi inclinée et la paroi verticale opposée sont en aluminium et elles sont maintenues à des températures constantes et différentes. Les deux autres parois sont en plexiglas. La méthode des ombres est utilisée pour visualiser le mouvement du fluide. L'apparition du mouvement cellulaire est tout à fait la même que dans le cas du chauffage d'une plaque verticale. Dans le régime de transition, des cellules de convection naissent successivement à partir de la base. Dans le régime supercritique il existe simultanément des cellules convectives. Ces cellules se développent dans le réservoir. Les mouvements dans les cellules complètement développées sont tout à fait différents de ceux observés dans des réservoirs à parois verticales. Le paramètre qui caractérise le mouvement du fluide est le nombre de Rayleigh défini à partir de la hauteur d'élévation d'une particule de fluide chauffé et de la gravité réduite. On trouve que le domaine du nombre de Rayleigh pour le régime de transition est plus grand que pour la paroi verticale. Ceci est du à l'effet stabilisant de la pression hydrostatique additionnelle normale à la paroi. Les tourbillons longitudinaux contre la paroi inclinée ont été observés seulement dans une cellule à grande base après six jours d'expérimentation. A cette époque le fluide dans la cellule était à salinité constante. Les tourbillons longitudinaux n'ont pas été observés dans d'autres cas.

ZELLENFÖRMIGE KONVEKTION IN EINER SALZLÖSUNG LÄNGS EINER BEHEIZTEN, GENEIGTEN WAND

Zusammenfassung—Es wurde die freie Konvektion längs einer beheizten geneigten Wand in einer Salzlösung experimentell untersucht. Die Versuche wurden in einem Behälter mit einer 45° geneigten Wand durchgeführt. Der Behälter war 20 cm hoch und ebenso lang, oben war er 30 cm breit, unten 10 cm. Die geneigte Wand und die gegenüberliegende senkrechte Wand waren aus Aluminium und wurden auf verschiedenen, konstanten Temperaturen gehalten. Die beiden anderen Wände waren aus Plexiglas. Zur Sichtbarmachung der Fluidbewegung wurde ein Schattenverfahren verwendet. Das Einsetzen der Zellbewegung ähnelt sehr dem Fall der beheizten senkrechten Wand.

Im unterkritischen Regime war die Flüssigkeitsbewegung längs der beheizten Wand kaum wahrnehmbar. Im Übergangsgebiet entstanden allmählich von der unteren Grenze ausgehend, Konvektionszellen Im überkritischen Regime war das gleichzeitige Auftreten von Konvektionszellen zu beobachten. Diese Zellen bildeten sich dann über den ganzen Tank aus. Die Bewegungen der voll ausgebildeten Zellen unterschieden sich stark von denen, die in Behältern mit senkrechten Wänden beobachtet wurden. Der massgebliche Parameter, der die Fluidbewegung charakterisiert, ist die Rayleigh- Zahl, gebildet mit der Steighöhe der erwärmten Fluidpartikel und der reduzierten Erdbeschleunigung. Es zeigt sich, dass der Bereich der Rayleigh- Zahl für das Übergangsgebiet etwas höher liegt, als für die beheizte senkrecht Wand. Das ist auf den stabilisierenden Einfluss des zusätzlichen hydrostatischen Druckes senkrecht zur Wand zurückzuführen. Längswirbel entlang der geneigten Wand wurden nur in einer grossen Zelle am Boden nach 6 Tagen Versuchsdauer beobachtet. In dieser Zeit war der Salzgehalt des Fluids in der Zelle konstant. In einem anderen Fall wurden Längswirbel beobachtet.

ЯЧЕИСТАЯ КОНВЕКЦИЯ ВДОЛЬ НАГРЕТОЙ НАКЛОННОЙ СТЕНКИ ПРИ НАЛИЧИИ ГРАДИЕНТА КОНЦЕНТРАЦИИ СОЛИ

Аннотация—Экспериментально исследовалась естественная конвекция вдоль нагретой наклонной стенки при наличии градиента концентрации соли. Эксперименты проводились в резервуаре, одна стенка которого имела наклон 45°. Высота и глубина резервуара составляли 20 см, ширина в верхней части—30 см и ширина в нижней части —10 см. Наклонная стенка и противоположная вертикальная стенка были изготовлены из алюминия и поддерживались при различных постоянных температурах. Две другие стенки были из плексигласа. Для визуализации движения жидкости использовалась теневая фотография. Ячеистое движение возникало аналогично случаю нагревания жидкости у вертикальной стенки. В докритическом режиме жидкость почти незаметно двигалась вдоль нагретой стенки. В переходном режиме конвективные ячейки образовывались последовательно от границы на дне резервуара. В сверхкритическом режиме наблюдалось одновременное образование конвективных ячеек, которые затем увеличивались в поперечном направлении. Конвективное движение в полностью развитых ячейках резко отличалось от движения, наблюдаемого в резервуарах с вертикальными стенками. Определяющим параметром, характеризующим движение жидкости, является число Релея, расчитанное по высоте подъема нагретой частицы жидкости и приведенной силе тяжести. Найдено, что диапазон изменения числа Рэлея в переходной области несколько выме, чем у вертикальной нагреваемой стенки. Это объястняется стабилизирующим влиянием дополнительного гидростатического давления, перпендикулярного к стенке. Продольные вихри вдоль наклонной стенки наблюдались только в большой ячейке на дне резервуара в течение 6 дней эксперимента. В это время концентрация соли в жидкости внутри ячейки была постоянной. Ни в каких других случаях продольные вихри не наблюдались.